Dimensions of the Circle of Willis and Dynamic Studies Using Electrical Analogy

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Many attempts have been made to study the dynamic changes in the circle of Willis under varying degrees of afferent arterial occlusion. These include ligational experiments,\textsuperscript{5,10,11} direct pressure measurements,\textsuperscript{2,20} resistance values in the carotid and vertebral arteries,\textsuperscript{6} studies of total cerebral blood flow,\textsuperscript{7,18,19} studies based on ophthalmodynamometry,\textsuperscript{10,16} and the use of glass or plastic models.\textsuperscript{1,8,14}

A fundamental prerequisite to any study of the dynamics of the circle is a knowledge of the dimensions of its components and their capacity to transmit blood. Without such a baseline it is unwise to speculate on the ability of the circle to act as an anastomotic channel in circumstances of acute haemodynamic change. Very few studies of this nature have been reported in the literature and none gives complete details of the length and calibre of all the components of the circle. This is perhaps not surprising, for it is now widely recognised that every circle is unique, and that there are no standard (normal) dimensions. Furthermore, the vessels are elastic and the flow in them is pulsatile. Both these factors make quantitative work still more difficult. Nevertheless it seems reasonable to assume that the absolute dimensions of the vessels as studied in the fixed state post mortem are in some degree related to the capacity of these vessels to transmit blood during life. Lewis\textsuperscript{9} and Metz\textsuperscript{12} made some measurements of the calibre of some of the component vessels, and further measurements of the length and calibre are reported in this present paper.

Because in the living animal the circle of Willis is difficult of access, and because it is still more difficult to measure directly any one of the parameters of pressure, volume of flow or peripheral resistance without distorting the system, a simple electric network analyser with inbuilt capacity for varying the capacity of flow of each component was constructed.

Such a system offers a more flexible means of studying variation in flow and pressure than do the glass or plastic models used previously, and though it takes no account of pulsatile flow or elasticity of the wall of the vessel these must be reducible to an average diameter and an integrated mean pressure. The analogue could then be set to the average found by actual measurement of the vessels of the circle, and the effect of varying degrees of occlusion of the main afferent components could then be studied.

Material and Technique

Dimensions of the Vessels of the Circle. Thirty-five brains in which the circle of Willis was complete were obtained from autopsies and fixed in formol-saline. The vessels were dissected off and the length of each component was measured. Numerous rings were then cut across the vessel and the unstretched diameter of the lumen was measured with a dissecting microscope fitted with a micrometer eyepiece. The diameter of each component was recorded as the average of 8 measurements of the diameter of each of the rings. The specimens primarily were separated into four age groups but statistical analysis showed no significant difference in the length or capacity of flow between the groups so grouping was abandoned.

Table 1 sets out the average length, diameter, capacity of flow (which is a function of the fourth power of the diameter divided by the length), average capacity of flow of the paired vessels and the percentage volume of flow through each of the main afferent and efferent vessels, this last being compared with the results calculated from the figures of Lewis.\textsuperscript{9}

Because it is commonly held that the circle of Willis is an equaliser as well as an anastomosis in the cerebral circulation, the paired vessels were
**TABLE 1**

*Dimensions and dynamics of components of circle of Willis*

<table>
<thead>
<tr>
<th>Vessel</th>
<th>Length mm.</th>
<th>Diameter mm.</th>
<th>D^2/L</th>
<th>Average D^2/L</th>
<th>Flow Capacity Ratio in Circle</th>
<th>Percentage Volume of Flow</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Present Series</td>
<td>Lewis^9</td>
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<td></td>
<td></td>
<td></td>
<td>Input</td>
<td>Output</td>
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<td>Left vertebral</td>
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<td>Right vertebral</td>
<td>2.250</td>
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<td></td>
<td>0.924</td>
<td>0.92</td>
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<td>8</td>
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<td>Proximal left posterior cerebral</td>
<td>7.3</td>
<td>1.475</td>
<td>0.644</td>
<td>0.62</td>
<td>10</td>
<td></td>
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<td>0.595</td>
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<td>Distal left posterior cerebral</td>
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<td>21</td>
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<tr>
<td>Distal right posterior cerebral</td>
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<td></td>
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<tr>
<td>Left posterior communicating</td>
<td>11.3</td>
<td>0.85</td>
<td>0.044</td>
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<td>0.875</td>
<td>0.052</td>
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<td>92</td>
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<tr>
<td>Proximal right carotid</td>
<td>2.625</td>
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<tr>
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<td>Proximal left anterior cerebral</td>
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<td>1.475</td>
<td>0.356</td>
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<td></td>
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<tr>
<td>Distal left anterior cerebral</td>
<td>1.400</td>
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<td>23</td>
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<tr>
<td>Distal right anterior cerebral</td>
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<td>1.000</td>
<td>2.00</td>
<td>0.2</td>
<td>28</td>
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</tbody>
</table>

* Average length and diameter of the components, with calculated capacity of flow (D^2/L), average capacity of flow, ratio of capacity of flow within the circle, and percentage of input and output volumes, the latter compared with results calculated from the measurements of Lewis.∞
examined for equality of diameter (within 5 per cent) and Table 2 shows that equality of diameter was seen 5 times more commonly in the efferent than in the afferent paired vessels. The distal part of the internal carotid artery also showed a greater frequency of equality than the other paired vessels, possibly because it sheds its asymmetric load via the posterior communicating vessels.

The Analogue. Fluid and electrical analogy is based on four fundamental characteristics which are illustrated in Fig. 1. (a) Flow of volume through a pipe and current through an electric circuit are measured by inserting a flow meter or ammeter in series with the system, (b) drop in pressure along a pipe and drop in voltage across a resistor are measured by inserting a meter in parallel with the system, (c) the amount of fluid or current approaching a junction equals the amount leaving that junction and (d) for flow to occur through both arms of a loop system the drop in pressure or drop in voltage in the clockwise direction must equal that in the anticlockwise direction.

These four fundamentals require only the use of linear resistors and form the basis of the analogue used by Camp and Hazen to analyse the system of water reticulation of a city. Some arithmetical correction, however, was required because in an hydraulic system the pressure-head loss varies nearly as the square of the rate of flow. McIlroy designed a nonlinear resistor to take account of this variation and made a direct-reading analogue for the same purpose as Camp and Hazen. The present analogue uses linear resistors and no arithmetical correction has been applied so that some slight inaccuracy is introduced. The analogue also simulates laminar flow which possibly is not correct for the system under study but seems a reasonable approximation in the light of our present knowledge.

The circuit diagram is shown schematically in Fig. 2, each unit consisting, as shown at the bottom, of a series of resistors mounted on an 11-position Yaxley switch in the position X1-X, each unit being led to a plug-board from which the current and voltage were measured on a university type ammeter with extended range to read 400 V. and 100 mA. Mains current was used transformed to 335 V. and half rectified with a 5Y3GT valve.

The resistors on the Yaxley switch are varied to comply with the required ranges obtained from the direct measurement of the volume of flow of each vessel (Table 1) converted to resistance units obtained from Fig. 3. R10 is a unit base load to prevent short-circuiting the whole system. R1-9 of the main afferent and efferent limbs are all of the same basic resistance and are used to obtain the correct ratio of input and output with a potentiometer at the origin for fine adjustment. Taking the distal part of the internal carotid artery to have a capacity of flow of 100 units with 1 base unit of resistance then the proximal part of the posterior cerebral artery requires the resistances to cover a range around 33 times this basic unit, the proximal part of the anterior cerebral artery requires some 64 times, the posterior communicating artery some ~56 times and the anterior communicating unit some 8 times the basic resistance unit. The final position of each unit is left unconf-
connected to provide for breaking of the circuit to simulate complete occlusion of the vessel.

A graph of the current through 1 unit of the analogue (actually the posterior communicating artery) is shown in Fig. 3 and is compared with the derived reduction in flow of fluid through a tube of fixed length with reduction of the calibre of the lumen, the calculation being simply the square of the reduced luminal area divided by the square of the original luminal area \( (A_r/A_o)^2 \).

Readings of current are converted directly into arbitrary units of flow giving a total of 100 to allow easy conversion when the normal total cerebral blood flow becomes known. Values of voltage are also converted to arbitrary values of pressure to facilitate transformation into percentage drop in pressure.

Only a few of the many possible experiments with the analogue will be described. Firstly, the only available figures against which the analogue could be matched are those of Rogers\(^1\) and of Avman and Bering.\(^1\) From the former paper only the results and a diagram are available, but no dimensions. It appears, however, that although the results were a little asymmetric the model was of uniform calibre except for the posterior communicating vessels which were smaller. The analogue was therefore set initially in this manner and the amperage and voltage readings were compared with Rogers’ figures, with the right carotid artery open and then completely occluded. Figs. 4 and 5 show these comparisons.

Avman and Bering’s model, although slightly more complex, is basically of the same design, and comparison of the changes in pressure and flow with small and large posterior communicating vessels falls consistently and systematically on either side of the results of the analogue, thus giving a very favourable comparison of the com-

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**Fig. 2.** Circuit diagrams of the analogue, each circle consisting of linear resistances mounted on Yaxley switches in the position X1-X of the bottom diagram. X is led to a plug-board from which voltage and current may be measured. R10 is a basic resistor of 1 unit and R1-R9 a series of linear resistors summing to give a value related to capacity of flow required.

**Fig. 3.** Comparison of current actually measured through one of the resistance units (actually a posterior communicating unit) and the derived reduction of flow of fluid through a straight tube of fixed length with reduced luminal area \( (A_r/A_o)^2 \).

**Fig. 4.** The proportional volume of flow through each component of the circle with the analogue set to simulate the glass model constructed by Rogers.\(^1\) The total flow in the original setting is taken as 100 units. Analogue open = ● — — — — ● Rogers’ model open = X Analogue right carotid occluded = O - - O Rogers’ right carotid occluded — ■
ponents measured and demonstrating further that pulsatile flow in an elastic vessel may, when considered over a period of time, be reduced to a mean without loss of accuracy.

If the capacity of flow of the distal part of the internal carotid artery is taken as 100 units, that of the proximal part of the posterior cerebral and anterior cerebral vessels and the posterior communicating vessels are approximately 10, 5 and 1 unit respectively (Table 1). If the lengths of the carotid arteries in the neck and of the vertebral-basilar system are taken as approximately equal, 29 per cent of the afferent flow enters the vertebral-basilar system and 35.5 per cent enters each carotid artery. Using the same approximation for the length of the efferent vessels to their finer ramifications, the value of outflow of each anterior, middle and posterior cerebral vessel is 9 per cent, 29 per cent and 12 per cent respectively.

The analogue was adjusted with the components of the circle of Willis set according to the values of capacity of flow derived by measurement and the afferent and efferent values derived by the approximation; and Figs. 6 and 7 show the diagrams of flow and pressure with the system open and with the right carotid artery occluded completely. The result of intervening stenotic positions can easily be filled in between these two extremes. The third record on these figures shows the effect of increasing the capacity of flow of the posterior communicating vessels by 25 times with the right carotid artery occluded completely.

The analogue, set in this manner and lacking the six cerebellar efferents and many smaller ones, is not suitable for the study of the effect of unilateral vertebral occlusion, and, as expected, the performance of this operation produced no observable alteration in the readings of flow or pressure distal to the occlusion.

Bilateral occlusion of the vertebral or basilar artery produces the same effect, and the alteration of flow and pressure within the system caused by this procedure is shown in Figs. 8 and 9 with the basilar artery open and then occluded completely. Again the intervening stenotic positions may be interpolated and the third record shows the effect of increasing the capacity of flow of the posterior communicating vessels to 25 per cent of the distal part of the internal carotid artery.

**Discussion**

Comparison of the changes in flow in the analogue with those in the glass model and the plastic model shows good agreement, but the electric analogue is more useful, not only because it is more flexible but also because it allows accurate measurement of changes in flow and pressure in all of the components of the system instead of only that of the efferent
vessels. This requires the measured values to be expressed in arbitrary units rather than the usual percentage change because in extreme cases, such as in the posterior communicating vessels, any change from the basic zero flow produces an infinite percentage change and in less extreme cases may still be misleading, especially in expressing volumes of flow. Conversion of the arbitrary figures of the analogue to percentage change is a simple subtraction and division.

Comparison of the changes in pressure with those of the plastic model of Avman and Bering\textsuperscript{1} also is very favourable, but those with the glass model of Rogers\textsuperscript{14} show several discrepancies. The records of pressure from the carotid arteries apparently have been transposed left for right, but, even allowing for this, the pressure in the right carotid artery distal to the occlusion is much higher than would be expected—in fact higher than the pressures recorded in the adjacent components through which this pressure must be exerted. This discrepancy is unexplained.

With steady flow of fluid through a rigid tube, a loss of 35 per cent of cross-sectional area results in a 12 per cent reduction in volume of flow (Fig. 3). Rogers\textsuperscript{14} found only a 41/2 per cent reduction in his model while the analogue showed a 14 per cent reduction.

When the effect of occlusion of the right carotid artery is studied in the analogue with the components set according to the anatomical measurements obtained, the most obvious effect is the gross reduction in flow of blood in the territory supplied by the occluded vessel, while the remainder of the tracts of outflow are little affected; in fact the contralateral hemisphere obtains a slightly increased supply (Fig. 6). The changes in pressure also show this extremely localised effect. Thus, if the analogue is a valid guide, the circle of Willis is a very poor anastomotic channel under acute conditions. Similar results obtain with complete occlusion of the basilar artery.

Since gross infarction of the middle cerebral territory by no means always follows occlusion of the internal carotid artery, the various possible ischaemia-modifying factors acquire added importance. Among these are:

1) an increase in blood pressure,
2) a decrease in peripheral resistance,
3) an increase in the calibre and thus in the capacity of flow of the various component vessels of the system,
4) peripheral cerebral and extracranial anastomoses, and
5) adequate cerebral nutrition can be maintained with a reduced cerebral blood flow.

Shenkin et al.\textsuperscript{18} have measured the mean arterial blood pressure in the femoral artery before and after unilateral ligation of the carotid artery and have shown an average increase of 20 per cent. However, although this theoretically would compensate for the reduction of the area of input if it applied to a single straight tube, it is not at all sufficient.
in a network of vessels when the capacity of flow of the components is considered. In fact, in 2 of the 4 patients reported by them, they also demonstrated a decrease of 22 and 25 per cent in the total cerebral blood flow (without neurological deficit in the first); the analogue under similar conditions showed a reduction of 30 per cent.

Many workers using direct cannulation or ophthalmodynamometry have shown an ipsilateral reduction in pressure distal to the site of occlusion of the carotid artery, but these findings allow us to infer little about changes in volume of flow.

Shenkin and Novack,19 using 5 per cent CO₂ inhalation, were able to demonstrate a reduction of the total cerebral vascular resistance of some 20 per cent. Reducing the peripheral resistance of the defrauded middle cerebral artery by 50 per cent after occlusion of the carotid artery on the analogue showed no measurable increase in flow to the area.

Enlargement of the posterior communicating vessels after occlusion of the carotid artery has been shown by Lowe10 and Cooper4 to occur in the rabbit. Lowe10 has extrapolated this finding to man, drawing support from the single autopsied case described by Cooper.4 However, I have been unable to find any record of actual measurement of the capacities of the intracranial or extracranial anastomotic channels in a patient with occlusion of the carotid artery. In the analogue, increasing the capacity of flow of the posterior communicating vessels from 1 to 25 per cent of the distal part of the internal carotid artery, which is equivalent to increasing the calibre of the posterior communicating vessel by 2.2 times the original diameter, failed to increase the flow to the middle cerebral territory beyond 30 per cent of its original level.

This failure of postero-anterior anastomotic flow via the posterior communicating vessels has been demonstrated by McDonald and Potter11 in rabbits subjected to unilateral occlusion of the carotid artery, and these animals have a much better developed posterior communicating artery than man. They presumed that the anastomotic flow under these circumstances occurred via the anterior channels from the contralateral carotid artery. Again, reducing the peripheral resistance of the ipsilateral middle cerebral vessel in association with unilateral or bilateral enlargement of the posterior communicating vessels increased the flow to the defrauded middle cerebral territory only from 10 to 11.5 units of flow compared with the original flow of 31 units.

In the absence of other reports 3 personally observed cases are of some relevance here. In the first there was bilateral chronic occlusion of the carotid artery in the sinus; in the second there was complete occlusion of one carotid and stenosis of the other to a diameter of 2 mm. in the sinus; while in the third there was bilateral stenosis of the sinus to 3 and 2 mm. In all 3 cases the calibre and capacity of all the anastomotic vessels were well below the average presented in this report.

Peripheral cerebral and extracranial anastomoses appear to be important, and those via the ophthalmic artery are being described with increasing frequency in the literature since the reports of Rosegay and Welch15 and Sachs.17 The two personally observed specimens with occlusion of the carotid artery and that described by Cooper4 showed the terminal half-inch of the extracranial carotid artery to be patent and it is possible that this preservation was ascribable to filling by backward flow through the ophthalmic-artery anastomosis. In fact, Woodhall et al.70 showed in 3 of their 4 reported cases that this anastomosis may be able to maintain in the isolated segment of the carotid artery, after intracranial and extracranial trap-ligation, a pressure equal to that commonly found distal to an acutely occluded carotid artery.

Considering now the ability of the brain to subsist on a reduced total cerebral blood flow, the normal range as estimated by the method of Kety and Schmidt7 is 54±12 ml./100 gm. of brain tissue/min. (i.e. ±22 per cent). This large range could be caused by gross inaccuracy of the method of measuring or gross variability in total cerebral blood flow, or a combination of both. Shenkin et al.18 also have demonstrated that patients
with a blood flow below 40 ml./100 gm./min., were all comatose but that no neurological deficit need occur with a reduction of 22 per cent of total cerebral blood flow after unilateral ligation of the carotid artery. Simple arithmetic thus indicates that those with an initial high cerebral blood flow may be able to withstand a total reduction of 40 per cent of total cerebral blood flow providing anastomotic and other mechanisms ensure that a basic minimum is still supplied to the defrauded area.

This minimum fortunately provides a reserve margin of safety for survival with only minimal assistance by the various physiological agents such as increased blood pressure, and reduced peripheral resistance and anastomoses for these mechanisms appear totally inadequate by themselves to avert the manifestations of cerebral ischaemia after occlusion of one of the main affenter vessels.

**Summary**

The length and calibre of the components of the circle of Willis have been measured and the relative capacity of flow of each has been estimated.

An electric network analyser has been constructed simulating laminar flow in rigid tubes, and the pressure and volume of flow in and through each component has been recorded under conditions simulating occlusion of the carotid and basilar arteries. The effects of increasing pressure or capacity of flow within the circle and of decreasing the peripheral resistance of the defrauded territory have been examined and have been considered insufficient to account for the relative infrequency of ischaemic sequelae subsequent to occlusion of a main affenter vessel of the circle.

Some of the literature on the subject has been reviewed and it is concluded that the ability of the brain to subsist on a reduced total cerebral blood flow, assisted by extracranial and peripheral cerebral anastomoses, plays a major part in preventing ischaemic sequelae after acute occlusive processes and possibly also after chronic occlusive processes, and the anastomoses forming the circle of Willis are of relatively slight assistance in this report.

I wish to thank Professor G. Newstead, now of the University of Tasmania, for the diagram of the circuit, the specifications and for helpful advice, and Professor J. S. Robertson for help and criticism in writing this paper.

**References**


